DIRAC AND HEISENBERG: CONGENIAL PARTNERS IN QUANTUM MECHANICS

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1. Introduction

In an evening lecture, given at the Trieste Symposium on Contemporary Physics in June 1968, Paul Dirac introduced the lecturer as follows:

> "I have the best of reasons for being an admirer of Werner Heisenberg. He and I were young research students at the same time, about the same age, working on the same problem. Heisenberg succeeded where I failed. There was a large mass of spectroscopic data accumulated at that time and Heisenberg found out the proper way of handling it. In doing so he started the golden age in theoretical physics, and for a few years after that it was easy for any second rate student to do first rate work."[1]

Four years later, on the occasion of celebrating Dirac's 70th birthday, also at Trieste, Heisenberg returned the compliment. In a lecture on the concepts of quantum theory, he addressed the relativistic theory of the electron and the subsequent prediction of the positron and then stated:

> "I think that this discovery of antimatter was perhaps the biggest jump of all big jumps in physics of our century. It was a discovery of utmost importance because it changed our whole picture of matter."[2]

In the first years of the 21st century we are commemorating the 100th birthdays of these two eminent physicists, who were congenial partners in developing the fundamental quantum-theoretical concepts, on which our present understanding of physics is based, and were at the same time dear friends throughout their lives.

To introduce my talk, let me first remind you of their curricula, which already indicate the possibility of quite a few interactions of the two.

Curriculum

	Werner Heisenberg		**Paul Dirac**
1901	5 Dec.: born in Würzburg, Germany of an academic family	**1902**	8 Aug.: born in Bristol, England; father Charles D. from Switzerland
1911-20	Student at the Maximilians-Gymnasium, Munich; *Abitur* with distinction	**1914-18**	Student at Merchand Venturer's Bristol
		1918-21	Studies at Engineering College, Bristol University
1920-23	Studies at Munich University, except for winter term 1922/23 at Göttingen (with Max Born)	**1921-23**	Studies in applied mathematics at Bristol University
1923	July: Dr.phil at Munich U. (with Arnold Sommerfeld)		
1923-24	Assistant to Born in Göttingen	**1923-26**	Studies in theoretical physics at Cambrige University (St.John's College); teachers: Ralph Fowler, Arthur Eddington, Edgar Milne, Henry Baker
1924	July: *Habilitation* at Göttingen		
1924-25	Rockefeller Fellow with Bohr in Copenhagen		
1925-26	*Privatdozent* in Göttingen	**1926**	May: Ph.D., Cambridge University
1926-27	Bohr's main assistant and lecturer at Copenhagen Univ.	**1926-27**	Research fellow with Bohr in Copenhagen and Born in Göttingen

1927-42	*Ordinarius* of theoretical physics at Leipzig University	**1927**	Fellow of St.John’s College
		1928	Attendance of the Soviet Physicists’ Conference
1929	Sabbatical and world travel to USA, Japan, China and India	**1929**	Lectures and world travel to USA, Japan and Russia
		1932-69	Lucasian professor, Cambridge
1933	Dec.: Physics Nobel Prize 1932	**1933**	Dec.: Physics Nobel Prize 1933
		1934-37	Several visits to Soviet Union
1937	Marriage with Elisabeth Schumacher, 7 children	**1937**	Marriage with Margit Wigner, 2 daughters
1939-42	Work for the German military uranium project (reactor construction)	**1942**	Work for the British military uranium project (isotope separation)
1942-45	Director at the *KWI für Physik* and professor at Berlin University		
1945-46	Internée at Farm Hall, U.K.		
1946-58	Director of the *MPI für Physik* in Göttingen	**1946-50**	Several visits to the USA and Canada
1949-51	President of the *Deutscher Forschungsrat*		
1952	Cofounder of *CERN*		

1953-75	President of the *A.v. Humboldt-Gesellschaft*	**1953-82**	Frequent visits of the Lindau Nobel laureates' conferences
1958-70	Director at the *MPI für Physik & Astrophysik*, Munich	**1968-71**	Extended visits of the *Center of Theoretical Studies*, Coral Gables
		1971-84	Professor at Florida State U., Tallahassee
1972	September: Celebration of Dirac's 70th birthday in a conference at the *CTP* , Trieste		
1976	1 February: H. died at Munich	**1984**	20 October: D. died at Tallahassee

From 1925 onwards they got into increasingly deep contacts, both personally and scientifically. They exchanged letters on physical problems, traveled together in 1929 round the world, and frequently met at the same conferences, e.g., the Brussels *Solvay Congresses* (1927, 1930, 1933, 1961) or the Lindau gatherings of the Nobel laureates (1953, 1956, 1959, 1962, 1965, 1968, 1971, 1973). From the very beginning of their exchange, they became close partners in science – e.g., a list of the rare references given by Dirac shows Heisenberg with 33 citations in the lead before Max Born with 27, Pascual Jordan with 25 and Wolfgang Pauli with 24 – and close personal friends, in spite of their very different characters: so, in contrast to Heisenberg, Dirac was quite retired and preferred to listen rather than talk.

2. Quantum Mechanics: Heisenberg's Discovery and Dirac's Fundamental Contributions (1925-1927)

At the origin of quantum mechanics there was the so-called "old quantum theory": i.e., the concept of the quantum of action, introduced by Max Planck's revolutionary step of 1900 into the theory of black-body radiation, and Niels Bohr's model of atomic structure of 1913, which Arnold Sommerfeld in 1915 extended to the Bohr-Sommerfeld theory. Since 1920 Sommerfeld and Bohr also acted as *the* decisive physics teachers of Werner

Heisenberg. In December 1922 their atomic theory experienced a last triumph through the discovery of the new element no.72, hafnium, which was predicted to be a homologue of zirconium rather than a rare-earth element; at the very same time Bohr received the Physics Nobel Prize. However, just a few months later Heisenberg and his third great teacher, Max Born of Göttingen, demonstrated in a thorough calculation that the Bohr-Sommerfeld theory failed to explain already the energy states of the two-electron atom helium. They concluded: “Either the quantum conditions are wrong, or the electron motions do not, even in the stationary states, satisfy the mechanical equations.”[3] The crisis of the “old quantum theory” aggravated in the following two years. As Dirac remarked in 1970 in looking back to this period:

> “Before 1925, people were working with Bohr orbits. Quite a lot of developments had been made about Bohr orbits. They provided a pretty satisfactory non relativistic picture of atoms in which one electron was the important one. ... But there were great difficulties in understanding how two electrons would interact. ... The young people in these days were trying to account for the spectrum of helium by setting up a theory of interaction of Bohr orbits, and there is no doubt that they would have continued along these lines if it had not been for Heisenberg and Schrödinger. They would have probably continued for decades.”[4]

In spring of 1925 Heisenberg achieved the crucial breakthrough to the modern quantum theory, quantum mechanics. Upon completing his paper entitled (in English translation) “On quantum-theoretical interpretation of kinematic and mechanical relations”[5], he traveled, on the invitation of the theoretician Ralph Fowler and the experimentalist Peter Kapitza, to Cambridge and presented there on 28 July 1925 a talk before the *Kapitza Club*, an association of the younger physicists there. Though Dirac was a member of this club, he later did not recall to have met Heisenberg at this occasion; on the other hand, he himself gave the talk at the following meeting. While Heisenberg had spoken in Cambridge on “Term zoology and Zeeman botanics”, a phenomenological description of atomic states, he did mention to Fowler his last work, who then asked him to send a copy of his publication on the new quantum mechanics, as soon as it might be available. Thus, when Heisenberg received the proof sheets of this in August, he dispatched a copy to Fowler, who turned it over to his “mathematically gifted” student Dirac for a closer look.

According to his later memories, Dirac set the paper away for a week or so, but then realized how important Heisenberg’s results were, which he then tried to relate to the familiar formalism of Hamilton-Jacobi variables. After some weeks he hit on the decisive point:

> “During a long walk on a Sunday, it occurred to me that the commutator [i.e., the difference $xy - yx$, where x and y denote two of Heisenberg’s noncommuting dynamical variables] might be the analogue of the [classical] Poisson bracket.”[6]

Thus, if p_r and q_r are a set of canonically conjugate pairs of variables (used in the classical description of an atomic system – e.g., the momentum und position variables), and x and y denote functions of them, then the following association exists between Heisenberg’s anti-commutators and the classical Poisson brackets:

$$x\,.\,y - y\,.\,x \leftrightarrow i\,h/2\pi \sum \left(\partial x/\partial q_r \cdot \partial y/\partial p_r - \partial y//\partial q_r \cdot \partial x/\partial p_r \right) \qquad (1)$$

From end of September to early November 1925, Dirac worked out a complete mathematical formulation of quantum mechanics based on the correspondence relation (1) and using mathematical quantities, which he called “q-numbers”. He then published the results in a paper, entitled “The fundamental equations of quantum mechanics”[7]. This formulation now was completely equivalent to the “matrix mechanics”, which Max Born, Pascual Jordan and Werner Heisenberg had developed slightly earlier in Göttingen, and Heisenberg wrote on 20 November 1925 to Niels Bohr:

> “Today I received a paper sent me by Dirac, in which he has done the mathematical part of the new quantum mechanics on the basis of my work (independently of Born and Jordan). ... In its style of writing some of it pleases me better than that of Born and Jordan.”

While Heisenberg in Göttingen continued to work on applying quantum mechanics to atomic problems, he watched eagerly the further progress of his English colleague. Thus he was “quite thrilled” by Dirac’s second paper dealing with the hydrogen atom, and he wrote to the author on 9 April 1926:

> “Your separation of the problem into two parts – calculation with ‘q-numbers’, on the one hand, and physical interpretation of the ‘q-numbers’, on the other hand – seems to correspond, in my opinion, completely to the nature of the mathematical problem.”

He expected that Dirac’s formulation might provide, besides the energy states of the hydrogen atom, also the intensities of the emitted spectral lines.

However, the new wave mechanics, developed and published meanwhile by Erwin Schrödinger in Zurich, solved this problem much more easily. While Heisenberg and Dirac agreed fully in rejecting Schrödinger's continuous interpretation of the wave-mechanical formalism, Heisenberg still hoped that "that the solution of the paradoxes in quantum theory "could be found in combining the Göttingen-Cambridge quantum mechanics with wave mechanics. Urged by him (in a letter of 26 May), Dirac studied in summer 1926 Schrödinger's theory more carefully and discovered, in particular, that the wave function of an atomic two-particle system can be written as either a symmetric or an anti-symmetric combination of two single-particle wave functions. For the symmetric case he derived a statistical behavior of the particles as follows from the relation of Satyendra Nath Bose and Albert Einstein (known since 1924); for the anti-symmetric case he obtained a different distribution of particle numbers N_s (having energy E_s) as a function of the temperature T, namely

$$N_s = A_s / [\exp(\alpha + E_s/k\,T) + 1], \tag{2}$$

with A_s denoting an expression depending on the state s and α a constant. Equation (2) applies to particles obeying Pauli's exclusion principle, as had been shown independently several months earlier by Enrico Fermi [8]. In a footnote on p.670 of his paper, submitted in August 1926, Dirac pointed out: "Professor Born has informed me that Heisenberg has independently obtained results similar to those" [9], referring to the latter's detailed calculation of the helium-atom states (submitted just a month earlier [10]). Soon afterwards he would meet Heisenberg in Copenhagen, and they stayed together with him for five months in a very fruitful scientific cooperation.

Dirac arrived in the middle of September 1926 at Copenhagen. In Bohr's institute, Heisenberg had assumed since May the positions of a main assistant and a lecturer at Copenhagen University. The main issue there had become the discussion of the interpretation of quantum mechanics, notably the debate on Schrödinger's interpretation of wave mechanics, which denied the existence of discontinuous quantum jumps. Thus Heisenberg first demonstrated, on investigating the quantum-mechanical fluctuation phenomena (submitted for publication in November 1926), that, when two identical atomic systems interact, one cannot speak of the energy as being transmitted between them continuously, so Schrödinger's interpretation demanded: of course, there existed also in quantum mechanics a certain analogy to the classical resonance phenomenon, in which the energy is passing from one vibrating system to the other (at a smaller beat frequency, but for any given state of the total system only time averages have physical significance [11]. This result would cast light on the relation between quantum mechanics and its experimental implications, which Dirac now studied in more detail. He claimed, in particular, that Heisenberg's conclusion was "capable of wide extensions", namely: "It can be applied to many dynamical systems, not necessarily composed of two parts in resonance with one another, and to any dynamical variable, not necessarily one that can take on only quantized values" – so he introduced his next paper, which he finished in early November 1926 [12]. In it he established a systematic quantum-mechanical transformation theory, which allowed him to demonstrate the one-to-one relation between the Göttingen-Cambridge *discontinuous* quantum mechanics and Schrödinger's apparently *continuous* wave mechanics. As a particular result, Dirac derived, by employing his singular "delta-function", the identity of the two main methods to calculate energy states in atomic systems: i.e., to diagonalize the energy matrix in the Born-Jordan-Heisenberg matrix formulation and to solve the eigenvalue problem of Schrödinger differential equation.

Simultaneously with Dirac in Copenhagen, Pascual Jordan in Göttingen developed a similar "statistical transformation theory", which avoided the explicit use of singular functions [13]. Heisenberg now went on, on the basis of the Dirac-Jordan results, to obtain in February 1927 his famous "uncertainty" or "indeterminacy relation", e.g.,

$$\Delta\, \mathrm{p} \cdot \Delta \mathrm{q} \;\geq\; (h\, /\, 4\, \pi), \tag{3}$$

which states that the precision of measurements of momentum *p* and position *q* is limited by the non-zero value of Planck's constant. He concluded in the abstract of his publication, submitted later in March:

> "This uncertainty is the real basis of the occurrence of statistical relations in quantum mechanics. Their mathematical formulation can be achieved by the use of the Dirac-Jordan theory. Departing from the foundation so achieved it will be shown how macroscopic processes can be understood from the quantum-mechanical point of view."[14]

That is, in less than two years after the breakthrough to quantum mechanics, Heisenberg worked out the essential point to complete its physical interpretation, a wonderful result also of a two-years cooperation with Dirac.

3. Dirac's Relativistic Electron Equation and the Dirac-Heisenberg Travel Round the World (1927-1929)

By the end of February 1927, before Heisenberg had finished the derivation of his uncertainty relation, Dirac went on from Copenhagen to Göttingen via Hamburg, where he discussed with Wolfgang Pauli the spin problem of the electron. Returning from Göttingen – there he had worked on the foundation of the electromagnetic field quantization – to Cambridge in summer 1927, he prepared the manuscript for his first lecture series [15]. The section entitled "The magnetic electron" (lectures 13-16) began with the remarks:

> "The quantum theory applied to electrons as point charges does not give results in agreement with experiment. It just gives half the number of stationary states per electron in an atom as are observed, and also it cannot account for the anomalous Zeeman effect."

Practically with the same words he introduced the following publication, received on 2 January 1928 by the Royal Society of London, in which he proposed a linear relativistic equation for the spinning electron (as he had developed in the lectures mentioned above) and obtained explicitly, at least to first order of accuracy, the relativistic correction of the hydrogen spectral lines, as well as the doubling of the number of states observed in atoms containing a valence electron (which had to have spin ½ in units of $h/2\pi$). In Part II of this work, submitted a month later, the author calculated successfully, in the same approximation, the relative intensity exhibited by the anomalous Zeeman effect of the atomic lines [16]. Soon after these publication appeared, Charles Galton Darwin of Edinburgh and Walter Gordon of Hamburg found the exact solution of Dirac's new equation for the hydrogen atom and confirmed the old (Sommerfeld) fine structure formula of 1916.

However, a serious problem, indicated by Dirac already in his first paper on the electron equation, remained unsolved: the occurrence of "electron" states with negative energies. Heisenberg, since October 1927 professor of theoretical physics at Leipzig University, remarked in a letter to Bohr (on 31 March 1928): " I have marvelled greatly at Dirac's works; but I find it very disturbing that such an apparently so complete theory as that of Dirac shows so terrible effects as transitions from positive to negative energy." He was very interested in the electron-theory situation, because he had collaborated since last fall with Pauli and Jordan on a relativistic generalization of quantum field theory, into which the new equation had to be incorporated. Therefore he invited the British friend to present his theory at the Leipzig "University Week" in June 1928. Dirac came indeed, but had little to offer with respect to the critical question except for the claim that the unwanted transitions might occur only very rarely. After some hiatus - they treated in 1928 other problems, such as ferromagnetism (Heisenberg) and the H-theorem (Pauli) in quantum mechanics – Heisenberg and Pauli finished their efforts to formulate relativistic quantum field theory without solving the "Dirac difficulties" [17]. "The work of Pauli and myself is at best a formal advance, perhaps not even that", Heisenberg admitted to Bohr on 1 March 1929, just before leaving Leipzig for an extended lecture tour to the United States, and he added: "I heard that Dirac will also be over there in the next months, so that I will be able to discuss theory a bit."

Indeed, already a year earlier in February 1928, Heisenberg had proposed to Dirac the idea of a joint tour round the world by writing (on 13 February 1928):

> "It is very probable that I will go to Chicago from April to September 1929; I have decided not to go this year; the journey next year is not decided yet. Of course I would be extremely glad, if we would work together those 6 months in Chicago and bring European life into the American hurry. If you go to Chicago, I will certainly go. Perhaps we could have some pleasure from seeing beautiful parts of the country, for example from seeing California, which I probably would visit in July or June. Or we would go back to Europe via Japan, India or China, etc. But of course you ought to do what you like best."

Three days after Heisenberg wrote this letter, on 16 February 1928, Arthur H.Compton sent a telegram to Dirac, stating: " Can you come next fall to winter. Heisenberg comes spring and summer." Dirac decided not to accept this offer, but agreed to visit in spring of 1929 the University of Wisconsin at Madison instead.

In January 1929, then, Heisenberg wrote again to Dirac: "I'll be in Cambridge, Mass., in March and in Chicago from beginning of April till the end of August. I am very glad to hear that you are going to America, too." Later in March 1929 he sent another letter from the American East coast to Dirac in Madison, announcing his soon arrival in Chicago, and continuing:

> "Since we are then not more than 200 miles apart from one another, I would like to establish the connection between us. Could you be so kind and to write me whether I may visit you some weekend, say April 6th or 13th. Then I would be interested to hear your opinion about Weyl's work. Weyl thinks to have the solution of the $\pm e$ difficulty. ... Before I left Germany, Pauli and I finished a paper about the relativistic

formulation of the many-body problem; I would very much like to show it to you and hear your opinion."

To be honest, not much scientific collaboration on the items envisaged occurred between Heisenberg and Dirac in the United States. Certainly, they met several times, also in Chicago and Wisconsin, but they spent much more of their time together on hiking tours in the Rocky Mountains. While Heisenberg lectured eagerly at the University of Chicago on "The Physical Principles of Quantum Theory" – so the title of the book emerging from those lectures [18] – and thus spread out the " 'Copenhagen spirit' of the quantum theory", his deserted collaborator Pauli in Zurich got together with the American visitor J.Robert Oppenheimer to improve their first formulation of "quantum dynamics of wave fields" – which was, as Oppenheimer recalled later, "in a ghastly state". They kept Heisenberg informed about their progress, who wrote back to Pauli on 20 July 1929, at the end of his Chicago time:

> "I find your investigations very nice, all the results seem to be very plausible, and the catastrophic self-interaction of the electron does not disturb me too much. ..., the theory will still change very much. Perhaps you can gain something for your interaction problem from my formulation without additional terms."

Here Heisenberg proposed, in particular, to apply gauge invariance in order to remove the ugly extra terms in the first Heisenberg-Pauli paper. (See their second communication on the relativistic quantum field theory [19].)

Dirac was also lecturing on quantum-mechanical topics at Madison. In fact, he presented there a course pretty much along the lines of his later, famous book *The Principles of Quantum Mechanics* [20]. At the same time, he and Heisenberg fixed their further travel to the West. Heisenberg, in particular, arranged the details of the voyage to Japan. Since the American ship was booked out, he selected a Japanese steamer leaving San Francisco on 14 August 1929. He wrote about it on 11 July to Dirac:

> "Our steamer is called Shinyo Maru and stops at Honolulu on August 20th. ... I will probably arrive in Berkeley August 12th and give three lectures there. On the way I will probably stop one or two days in Yellowstone Park. Are you going there, too?"

Dirac indeed did, and Heisenberg recalled later a story about their getting together – they arrived there separately – in the spectacular American National Park, which characterizes Dirac quite a bit:

> "We agreed to meet at the hotel at the Old Faithful Geyser, and I had written to Paul that it would be nice if we could look at some of the geysers going off just at the moment they go off. When we met I heard that he had made out a very careful time table of all the geysers which one could possibly reach at the time of their going off, and he also wrote out the distance from one geyser to the next. Thus we could walk over in such a way that by a very carefully selected trip we could actually see quite a number of them going off."[21]

Before leaving America, Heisenberg and Dirac spent two days at the University of California in Berkeley lecturing on quantum-mechanical topics, and Heisenberg in addition spoke on the relativistic quantum field theory. On their way to Japan, they stopped, as previously planned, at Honolulu for a sightseeing tour with local physicists, and then they went on to arrive on 30 August 1929 in Yokohama.

The invitation to visit Japan came about this way. Back in fall of 1928, on his way home after a several-years' stay in Europe (notably with Ernest Rutherford in Cambridge and Niels Bohr in Copenhagen), Yoshio Nishina of the *Riken* Institute in Tokyo had passed through Cambrige, England, and made plans with Dirac to visit Japan. Nishina was very interested in the relativistic electron theory, as he had just worked out with Oskar Klein in Copenhagen the problem of scattering of light by electrons in atoms and obtained an experimentally well substantiated formula, which took into account Dirac's negative-energy states [22]. He received Dirac and Heisenberg at Yokohama harbor and accompanied them to their lectures in Tokyo, September 2-7, and later at Kyoto Imperial University [23]. Among those, who attended these lectures on the latest quantum-mechanical results of the European pioneers, were the two third-year students Sin-itiro Tomonaga and Hideki Yukawa from Kyoto. Tomonaga later recalled the impact of these presentations:

> "On the last day of the lectures at the University of Tokyo, Professor Nagaoka got up and raved about how Heisenberg and Dirac in their twenties had accomplished such a major thing as the establishment of a new theory and deplored that in Japan the physicists were still picking up the chaff and bran of Europe and America, and that students [in Japan] were just copying lectures, which was terrible. 'You guys should emulate Heisenberg and Dirac.' "[24]

Well, both Tomonaga and Yukawa followed Nagaoka's demand within a few years, and they became very productive along the paths shown by the famous visitors from Europe.

Besides lecturing, Dirac and Heisenberg enjoyed seeing some beauties of their host country: e.g., they visited the famous temples at Nikko. Dirac reported the following incident that was very characteristic for his friend:

> "In Japan, I found how good he is at mountaineering and what wonderful sense of height he has. We had to climb a high tower with a platform at the top, surrounded by a stone balustrade. At each of the four corners the stone-work was a little bit higher. Heisenberg climbed up on the balustrade and then on the stone-work at on of the corners and stood there, entirely unsupported standing about on six inches square of stone-work. Quite undisturbed by the great height, he just surveyed the scenery around him. I could not help feeling anxious." [25]

From Japan they both returned to their respective home universities, though on different paths: Heisenberg traveled mainly on sea via China (Shanghai) and India (Calcutta), while Dirac took the Trans-Siberian train to Moscow on his way to England.

4. Holes, Nuclear Forces and Elementary Particles (1930-1965)

Having returned from his world tour, Dirac began to think again about his four-component electron equation, of which only two were associated with positive- energy states and seemed to have physical meaning, although the negative-energy states occurred in all calculations dealing with physical processes. In a series of lectures, which he presented in December 1929 in Paris, he followed the idea expressed by Hermann Weyl and claimed that the problematic states behaved somehow like protons, if he made "the following hypothesis that almost all negative energy states in the universe are occupied". As he wrote to Bohr on 26 November 1929, the "few electrons of positive energy [existing in the real world] will be unable to make transitions to states with negative energy and will therefore behave quite properly", while "the distribution of negative energy electrons will be of infinite density, but quite uniform so that it will not produce any electromagnetic field, and one would be unable to observe it". Thus he introduced what was later called the "Dirac sea", interpreting the distinct holes (due to Fermi statistics) in it as protons [26].

One and half year later – under the impression of various arguments expressed by Weyl, Igor Tamm, Robert Oppenheimer und Wolfgang Pauli and others – Dirac abandoned the identification of holes with protons and rather postulated the existence of a "new particle, unknown to experimental physics, having the same mass and opposite charge of an electron". "We call such a particle an anti-electron", he declared, and predicted further that this new, stable particle could be produced in a high vacuum by an "encounter between two hard γ-rays (of energy at least half a million volts)", leading "simultaneously to the creation of an electron and an anti-electron" [27]. While in April 1932 the participants at the Copenhagen meeting discussed in a jocular version of Goethe's *Faust* the many problems of Dirac's electron theory [28], Carl Anderson at Pasadena announced in August 1932 that he and Seth Neddermeyer had detected in cosmic-ray tracks (taken with a cloud chamber operating in a strong magnetic field at high altitudes) "a positively charged particle comparable in mass and magnitude of charge with an electron" [29]. In February 1933 Patrick Blackett and Giuseppe Occhialini of the Cambridge Cavendish Laboratory confirmed that "negative and positive electrons may be born in pairs during the disintegration of light nuclei" [30], exactly as Dirac had predicted two years ago.

The twin production of electron-positron pairs – the name "positron" the Americans had given to the anti-electron – persuaded most previously skeptical colleagues to take Dirac's "hole theory" seriously. So Heisenberg wrote on 9 October 1933 to Arnold Sommerfeld:

> "What do you say about positive electrons? It seems indeed that Dirac is much more right than we thought so far. I spent the past weeks in Copenhagen and talked much with Dirac about the hole theory. Until now it still presents problems to separate the hole theory from the problems which one cannot yet solve with today's theory: [to calculate the] value of e^2/hc, the electron's radius, etc. However, for a number of questions: Meitner-Hupfeld effect, bremsstrahlung, etc., the theory seems to suffice. Even Bohr was very enthusiastic about it."

A month later he met Dirac again, this time in the castle of the Swedish king in Stockholm, where Heisenberg received the Physics Nobel Prize of 1932 for his discovery of quantum mechanics, and Dirac shared with Schrödinger the Nobel Prize of 1933 for developing fruitful schemes of the modern atomic theory. Unlike Pauli, who called the whole scheme "Dirac's Commandments of the Law of Nature on Mount Sinai" and still remained opposed to it, Heisenberg started to apply the hole theory to treat various problems of quantum electrodynamics that could be evaluated by applying proper care with the occurring infinities: e.g., the "charge fluctuations" due to "virtual" electron-positron pairs [31], or – in collaboration with his student Hans Euler – the nonlinear corrections to the Maxwell equations in vacuum [32]. This work, together with Dirac's pioneering papers, started today's "renormalized quantum electrodynamics", although Dirac tried in the later 1930s a different path by modifying the (classical) interaction of the electron with the radiation field, such as to remove the infinite Coulomb self-energy [33].

Besides the work on Dirac's hole theory, Heisenberg pursued with Pauli the attempts to create a completely finite quantum electrodynamics, in which also the fine structure constant could be calculated. Further he worked in the 1930s on different burning topics of the day. First, he inaugurated, after the discovery of the neutron in early 1932, the modern theory of the atomic nucleus, just on the basis of the usual quantum mechanics and the introduction of a new type of exchange forces: these nuclear forces, bound the constituent protons and neutron together in the nuclei [34]. Then, after Enrico Fermi's successful theory of beta-decay, Heisenberg tried to attack, since early 1934, his and Pauli's great vision: namely, a "unified theory" of *all* elementary particles and their binding forces on the basis of the so-called "Fermi-field theory", which not only seemed to account – at least for a couple of years – for *all* nuclear phenomena, but perhaps (via Louis de Broglie's 1934 idea of the neutrino theory of light) also for the electromagnetic ones; and finally, it offered the possibility to explain new interactions in cosmic-ray processes at the highest energy [35]. Yet Pauli was very skeptic towards it because of the serious divergence problems occurring there, while Dirac stayed neutral. However, a young Japanese, Hideki Yukawa – the same who had listened in 1929 as student to Dirac's and Heisenberg's lectures – came up (already in fall of 1934) with the idea to explain the Heisenberg's nuclear exchange forces in a different way, i.e., by postulating the existence of a further, new particle, the "heavy quantum" that should have intermediate mass between the proton and the electron [36]. When in 1937 a particle of about these properties was found (by Anderson and Neddermeyer and others in cosmic-ray processes (they called it the "mesotron"), the Fermi-field theory had to be abandoned. On the other hand, Heisenberg now succeeded with the instable mesotron to solve (together with Hans Euler) a great riddle observed of cosmic radiation, the existence of the so-called "hard component" [37].

Thus, by the end of the 1930s a new field of quantum physics, high-energy nuclear physics or elementary particle physics, had emerged, based especially on the fundamental concepts contributed by Dirac, Heisenberg, Pauli and Yukawa. However, at the same time tremendous difficulties showed up in establishing a consistent theory. Their removal certainly required further, yet unknown, revolutionary concepts. Heisenberg suggested one in 1938, the idea of a "smallest length", but this step did not lead to a detailed field-theoretical scheme. Thus he suggested, in the middle of World

War II, a alternative approach, and he described elementary particles and their interactions by the so-called “scattering matrix” or “S-matrix”. He based the scheme on the same philosophical principle that had guided him in 1925 to quantum mechanics – i.e., to use only observable quantities in the theoretical formulation [38]. The S-matrix attracted in the years following the war quite some interest in the scientific community, and Dirac sent one of his Ph.D. students, Richard Eden, from England to Heisenberg in Göttingen to learn the topic from the master. Meanwhile Dirac had him also come out with a different idea to solve the divergence problem in relativistic quantum field theory. In his Bakerian Lecture of 19 June 1941, Dirac suggested, in particular:

> “The simplest way of developing a theory would make it to apply to a hypothetical world in which the initial probability of certain states is negative, but transition probabilities calculated for this hypothetical world are found to be always positive, and it is, again, reasonable to assume that these transition probabilities are the same as those for the actual world.”[39]

That is, he suggested to keep the standard formalism of quantum field theory, however, to apply simultaneously a new interpretation involving also negative probabilities, and this procedure then would permit at least the removal of some of the crucial infinities.

Pauli, the severe critic of many of Heisenberg’s new ideas, found that Dirac’s “indefinite metric” field theory was problematic as well, and finally would run into serious troubles, leading to negative probabilities for some physical processes. On the other hand, it helped a little to formulate an explicitly covariant scheme of quantum electrodynamics, in which the appearing negative-energy “ghosts” could be associated with unphysical longitudinal and scalar photons. However, in 1955 Heisenberg picked up Dirac’s indefinite-metric quantum field theory to remove radically all divergence difficulties in his new, unified spinor theory of all elementary particles, which he developed partly together with Wolfgang Pauli, and which led in early 1958 to the so-called “world formula”. While Pauli’s soon criticized mathematical and physical details of the theory – unfortunately, he died soon in December 1958 – Heisenberg went on to develop further the unified field theory. Several years later he claimed to have achieved a particular triumph, as he wrote to Dirac on 19 January 1965:

“I am looking forward to our meeting in Lindau next summer. I am sending you a preprint about the famous coupling constant e^2/hc which we have discussed so much in old times. As you will see from the paper, in the end the calculations are quite simple and the numerical result is quite satisfactory. The coupling constant is actually determined not within quantum electrodynamics alone but by connecting quantum electrodynamics with a general theory of elementary particles. I am convinced that this is the final solution in spite of unavoidable shortcomings of approximation methods."

5. Conclusion

Later than 1930, Heisenberg and Dirac never had been together for a longer period together at the same place. During World War II, they found themselves in enemy camps, although the separation never was complete – as Heisenberg had some contact via Dirac's brother-in-law, Josef Tezler in Hungary, whom he could even assist a little in personal problems. After the war, especially since the 1950s, they began to meet again at many occasions, notably at the Lindau meetings (as we have mentioned already earlier) to exchange ideas. They were normally seen to stick together like twins. This did not imply that they always agreed fully in their opinions. Thus Dirac raised the following objection to Heisenberg's last unified theory of elementary particles (in a letter of 6 March 1967):

> "I do not think that your basic equation has sufficient mathematical beauty to be a fundamental equation of physics. The correct equation, when it is discovered, will probably involve some new kind of mathematics and will excite great interest among the pure mathematicians, just like Einstein's theory of the gravitational field did (and still does). The existing mathematical formalism just seems to me inadequate."

It was the mathematical beauty, which had guided Dirac to his greatest success, the relativistic electron equation – which, by the way, also served Heisenberg, as the basis of his nonlinear spinor theory. Dirac further believed that perhaps the electron played a special role in building up matter. Heisenberg knew that Dirac always felt that "*one can only solve one difficulty at a time*", he himself stuck to the opposite opinion, namely: " *You can never solve only one problem at a time, you have to solve always quite a lot of difficulties at the same time*." [40] After all, this was the philosophy behind his own greatest success, the discovery of quantum mechanics.

However, it was perhaps exactly their different philosophical attitudes in creating new scientific ideas that had contributed most to their fruitful interaction. We may finally add that they shared the same reservation with respect to a successful method of quantum field theory, which was based their own work. Thus Dirac, after the death of his partner, stated clearly:

> "We should no longer have to make use of such illogical processes as infinite renormalization. This is quite nonsense physically, and I have always been opposed to it. … In spite of all successes one should be prepared to abandon it completely and look on all successes hat have been obtained by using the usual forms of quantum electrodynamics with the infinities removed by artificial processes as just accidents when they give the right answers, in the same way as the successes of Bohr's theory are considered merely as accidents when they turn out to be correct." [41]

Was this just a prejudice of aged men, who had been geniuses in the Golden Age of theoretical physics?

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The talk has been based on an article, written together with Laurie M.Brown (Paul Dirac and Werner Heisenberg – a partnership in science) and published in the Dirac memorial volume, *Reminiscences About a Great Physicist*, edited by B.N.Kursunoglu and E.P.Wigner, Cambridge University Press, Cambridge 1987, pp.117-162. Additional material collected since the time of this publication has been explored in two books, namely in J.Mehra and H.Rechenberg's *The Historical Development of Quantum Theory*, Springer-Verlag, New York 2000-2001, and in L.M.Brown and H.Rechenberg's *The Origin of the Concept of Nuclear Forces*, IOP, Bristol-Philadelphia 1996.

The author would like to dedicate this talk to Laurie Brown, Evanston, for nearly two decades of a wonderful fruitful collaboration on the historical development of nuclear and elementary particle theory

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