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## SOME EPISODES OF THE HIDDEN HISTORY OF A NON TRADITIONAL INTERPRETATION OF THE THEORY OF GENERAL RELATIVITY

### 1. M. Friedman

The non traditional interpretation to which we make reference is explained in the well-know work of M. Friedman, *Foundations of Space-Time Theories*. We concentrate on the principle of covariance, on that of equivalence, on the connections along them and on those between them and the general principle of relativity. We believe that a short and schematic summary of Friedman's work, led from this point of view, will facilitate the comprehension of this work.

We know that Einstein's principal motivation in developing the general theory of relativity was to eliminate all absolute motion and to fully implement a relativistic conception of motion. He wanted to eliminate absolute acceleration which was not being cancelled by the special theory of relativity. Moreover, he wanted to make it including the effects of the gravitational field. One of the principal ideas to achieve this purpose was to eliminate the privileged class of inertial reference frames and to formulate laws of motion valid in arbitrary reference frames: these laws were called *generally covariant*. «The point, of course, is that such generally covariant laws appear to implement a thoroughgoing "equivalence" [...] of all states of motions and to extend the classical and special principle of relativity to a *truly* general principle of relativity.»<sup>2</sup>

We can now argue simpler about the two principles if we make a conceptual distinction between them.

#### 1.1. Principle of Covariance

Friedman distinguishes between intrinsic and extrinsic features of a space: «intrinsic features characterise the geometrical structure of the surface - its curvature, Euclidean or non-Euclidean character, and so on - and are completely independent of any particular coordinatization of the surface. Extrinsic features, on the other hand, correspond to particular coordinatizations of the surface; accordingly they vary as we change from one coordinate system to another.»<sup>3</sup> A theory will be *generally covariant* just in case it can be given an intrinsic, or coordinate-independent, formulation for it. But what is the meaning of 'coordinate-independent formulation'? In the coordinate independent formulation, the objects of the theory are seen like various kinds of abstract maps. The space-time of the general theory of relativity effectively i) has a topology: given any point  $p$  in space-time, one has the notion of a neighbourhood of  $p$  - a set of points all of which are "close" to  $p$  and ii) is coordinatizable by  $R^4$  - the set of quadruples of real numbers. That is, given any point  $p$  in space-time, there exists a neighbourhood  $A$  of  $p$  and a one-one map  $\mathcal{O}$  from  $A$  into  $R^4$  that is sufficiently continuous.  $\mathcal{O}$  is called a coordinate system, or chart, around  $p$ . Such a chart enables to translate statements about geometrical entities in space-time into statements about real numbers. In effect, the coordinate-independent formulation takes into consideration tangent vectors, which are mapping from real valued functions to real numbers, the affine connections, which are mapping from vector fields to vector fields and the metric tensors, which are mapping from pairs of vectors to real numbers. «The equation of a space-time theory  $T$  pick out a class of dynamically possible models  $\langle M, \gamma^1, \dots, \gamma^n, T \rangle$  - where  $M$  is a four-dimensional manifold;  $\gamma^1, \dots, \gamma^n$  are the geometrical objects postulated by  $T$ ,  $T^{\wedge}$  is the tangent vector field to a class of curves  $\hat{\gamma}$  on  $M$ ;  $\gamma^1, \dots, \gamma^n$  satisfy

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<sup>1</sup> I am deeply indebted to Silvio Bergia for his comments on early presentation of this work and for his assistance in writing the final version.

<sup>2</sup> See [4], p. 17. Italics in original.

<sup>3</sup> See [4], p. 9.

the field equations of  $T$ ; and  $T^*$  satisfies the laws of motion of  $T$  - and the class of models picked out by the theory is independent of the choice of any particular coordinatization of  $M$ .»<sup>4</sup>

However, since the requirement of covariance is just the requirement that a theory can be formulable in the intrinsic, coordinate independent style, then it will be satisfied by any theory which can be formulated in this way. Friedman shows this is possible in reference to both the Newtonian gravitational theory and the special and the general theory of relativity. Therefore Friedman can affirm that «the principle of general covariance has no physical content [...]: it specifies no particular physical theory; rather, it merely expresses our commitments to a certain style of formulating physical theories.»<sup>5</sup> In conclusion, this reason allows us to think that the covariance has nothing to do with the relativity of motion and namely with the general principle of relativity. Indeed the latter, rather than the former, has physical content and meaning and the principle of general covariance does not correspond to the notions of physical equivalence and relativity.

### 1.2. Principle of Equivalence

The meaning of the principle of equivalence is the following: accelerating frames are indistinguishable from non accelerating frames in the presence of gravity. But for Friedman the principle of equivalence can be better understood if we describe the gravitation in terms of a non flat space-time in which gravitational trajectories follow the geodesics and there is not gravitational force: freely falling particles follow inertial (geodesic) trajectories (local inertial frames). In Einstein's approach the principle of equivalence works as an argument for the general principle of relativity: an extension of the special principle of relativity which holds for uniform motions to cover non uniform, accelerated motions as well. However Friedman has showed that a dynamical space-time theory of gravitation does not automatically relativize acceleration in the way that the special theory of relativity relativizes velocity.

Einstein's point of view is the following: in conventional Newtonian gravitation theory (without global boundary conditions) it is impossible to distinguish an inertial reference frame  $K$  from an arbitrary accelerating reference frame  $K'$ . In conclusion, the theory satisfies the following version of the principle of equivalence:

(E) *All reference frames are physically equivalent or physically indistinguishable,*

where «two reference frames are equivalent just in case no "mechanical experiment" can distinguish between them.»<sup>6</sup>

«In a space-time theory ( $T$ ),

(R2) *if two reference frames are indistinguishable according to  $T$ , they should be theoretically identical according to  $T$ .*»<sup>7</sup>

Since  $\langle R1 \ \& \ R2 \rangle$ , where R1 affirms that *all inertial frames are physically equivalent or indistinguishable*, lead to the special principle of relativity (the relativization of velocity), Einstein thought that  $\langle E \ \& \ R2 \rangle$  could in fact led to something like a general principle of relativity, that  $\langle E \ \& \ R2 \rangle$  does for acceleration what  $\langle R1 \ \& \ R2 \rangle$  did for velocity. But this line of thought is misleading: rotating reference frames are physically distinguishable from non rotating reference frames. Summarizing, this line of thought does not led to a

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<sup>4</sup> See [4], p. 48.

<sup>5</sup> See [4], p. 55.

<sup>6</sup> See [4], p. 150.

<sup>7</sup> See [4], p. 153.

complete indistinguishability between all the arbitrary moving reference frames and thus does not satisfy the general principle of relativity.

Hence, if we interpret the principle of equivalence as (E), this principle is not actually satisfied of the general theory of relativity. But Friedman considers three further interpretations.

(1) *The equivalence of Gravitational and Inertial Mass.*

All bodies move in the same way in a gravitational field regardless of their different masses.

The principle of equivalence needs only apply to the two concepts of i) inertial mass and ii) gravitational mass:  $m_i = m_g$  is sufficient for the result that all bodies follow same trajectories in a gravitational field. In fact, the equality of inertial and gravitational field, from Friedman's point of view, allows us to reconstruct the world-lines of gravitationally affected particles as geodesics of a non flat connection. It implies the existence of a connection  $\overset{\circ}{D}$  such that freely falling bodies follow geodesics of  $\overset{\circ}{D}$ . Hence, the principle of equivalence, from the point of view of this interpretation, must be true if any theory of gravitation like general relativity, in which gravitational interaction is explained by the dependence of a non flat connection on the distribution of matter, is to be possible. And, of course, general relativity is not the unique theory of this sort: for example, classical gravitation theory can also be formulated in this way.

A possible way to distinguish these two theories is to further suppose that the non flat connection  $\overset{\circ}{D}$ , whose existence is guaranteed by the equivalence  $m_i = m_g$ , must also be unique.

(2) *The Uniqueness of the Gravitational Connection.*

The non flat connection  $\overset{\circ}{D}$  determined by the freely falling trajectories is unique.

Hence, there is no distinguishable flat connection  $\overset{\circ}{D}$ , and there is no distinguishable class of inertial frames following the geodesics of  $\overset{\circ}{D}$  (where  $\overset{\circ}{D}$  is a non flat connection analogous to the connection of the formulation of the Newtonian theory, which does not incorporate the gravitational potential into the affine structure, and where the fixed back-ground space-time structure of this theory is just Galileian space-time<sup>8</sup>). There are only local inertial frames following the geodesics of  $\overset{\circ}{D}$ .

This version of the principle confers significance to Einstein's often repeated claim that general relativity explains the equivalence of inertial and gravitational mass in a way that Newtonian theory does not.<sup>9</sup>

But replacing the inertial frames following the geodesics  $\overset{\circ}{D}$  with the local inertial frames of  $\overset{\circ}{D}$ , Friedman clearly shows that the latter are all equivalent to one another but not to arbitrary accelerating frames of  $\overset{\circ}{D}$ , while the inertial frames of  $\overset{\circ}{D}$  are equivalent to arbitrary accelerating frames of  $\overset{\circ}{D}$ <sup>10</sup>. Although in this way there are no inertial frames in general relativity, there are inertial

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<sup>8</sup> Cf. [4], p. 95.

<sup>9</sup> The equality between  $m_i$  and  $m_g$  is necessarily integrated with other important facts and the connection  $\overset{\circ}{D}$  plays a role in the explanation of other phenomena (for example, electrodynamical phenomena). In the Newtonian theory (in the formulation that does not incorporate the gravitational potential into the affine connection) this equality is purely accidental. Cf. [2].

<sup>10</sup> Cf [4], III, 3 and III, 4, pp. 92-104.

(geodesic) trajectories, and these trajectories give rise to an absolute distinction between inertial and non inertial (accelerating or rotating) motion, just like in special theory of relativity. It is not true, therefore, that all frames of reference are "equivalent": there exists a privileged subclass of frames, the local inertial frames, and the existence of such a subclass clearly shows that the general theory of relativity does not institute a thoroughgoing relativity of motion. Concluding, for this reason the principle of equivalence does not eliminate privileged reference frames and privileged states of motion.

*(3) The "Local" Equivalence of Freely Falling Frames and Special Relativistic Inertial Frames.*

The law of motion characterising the local inertial frames takes the same form of the law of motion characterising the inertial frames of the special theory of relativity, but only along the trajectory defined by the origin of the frame. But this third version of the principle of equivalence is strictly false if "local" has the usual mathematical meaning: on some small (but finite) neighbourhood. If there is a non vanishing gravitational field at a point  $p$ , then  $K = 0$  at  $p$  (where  $K$  is the curvature tensor of the connection  $D$ ) and there is no neighbourhood of  $p$  in which  $R^i_{jk} = 0$  (where  $R^i_{jk}$  represents the components of  $D$ ). In other words, freely falling frames are only "infinitesimally" equivalent to inertial frames: only at a single point or on a single trajectory. For this reason gravitational forces cannot be equated with "apparent" or "fictitious" inertial forces and, therefore, the general principle of relativity cannot be justified from the principle of equivalence.

**2. E. Kretschmann and the principle of general covariance**

Erich Kretschmann has been often cited like the first (1917) who recognised that the general covariance of the equations of Einstein's general theory of relativity does not necessarily entail that the theory satisfies a general principle of relativity: general covariance can be always satisfied by any equation through a mathematical re-formulation using the absolute differential calculus of Ricci and Levi-Civita. Since it is still true for equations representing non-relativistic laws of nature, the principle of general covariance does not represent the mathematical structure of the principle of general relativity and, moreover, the latter, in reference to Einstein's formulation, is devoid of physical content. This is what is normally thought of Kretschmann's contribution.

The main purpose of Kretschmann was to demonstrate that the theory of general relativity is not a relativity theory but an absolute theory and to justify this purpose, he wrote the above considerations about the principle of general covariance in the beginning of his article. He arrived at his goal through a long and obscure demonstration in which the most difficult problem is represented on the lack of distinction between the concepts of 'reference system' and 'coordinate system'. The method used by Kretschmann is to find a way to endow the special principle of relativity with a physical content and then to show that, through the same way, the principle of general relativity cannot be endowed with a physical content. The way to find a physical content of a relativity principle is the following:

1) with each equation expressing a law of nature is associated a geometric four-dimensional picture <sup>11</sup>;

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<sup>11</sup> These "geometric pictures" are coordinate-dependent representations and not intrinsic models.

2) the same law can be represented on many equations depending on the choice between the coordinate systems. All the equations representing the same law can be formulated in a generally covariant form;

3) with the covariant mathematical expression of the same law is associated a geometric picture;

4) through the comparison of all the pictures, one obtain a set of those which show the same catalogue of topological coincidences of events-points (point-coincidence argument) and hence a distinguishable set because of its observability;

5) since to each picture corresponds a coordinate system and since for each coordinate system is valid a transformations group, then to the set of pictures corresponds a set of transformations groups;

6) the common characteristics of the transformations groups which constitute the set, for example the number of parameters, excluding the characteristics depending on the choice of reference frame<sup>12</sup>, determine the invariance group  $G$  which is univocally associated with the relativity postulate.

In the final analysis the relativity postulate is endowed with a physical content, that is, univocally associated with a class of geometric four-dimensional pictures, through the invariance group  $G$ .

As an example Kretshmann takes the usual non-generally covariant formulation of the law of light propagation:

$$(x_1 - x_1^0)^2 + \dots + (x_4 - x_4^0)^2 = 0,$$

and its generally covariant formulation:

$$ds = 0,$$

$$ds^2 = g_{\mu\nu} dx_\mu dx_\nu = 0,$$

$$(\mu, \nu) = 0 \quad (\mu, \nu = 1 \dots 4).$$

The conclusion is that the covariant group of the generally covariant formulation is wider than the relativity group satisfied by the law, «Since the additional covariance serves only to introduce families of geometric pictures that are representationally redundant. In this sense, the generally covariant formulation can be reduced to the less covariant formulation.»<sup>13</sup> This conclusion is possible because, in special relativity, the relativity group arises by imposing non-covariant constraints on the generally covariant formulation, without changing the physical content of the theory, in order to arrive at an non-generally covariant formulation, whose covariance group coincides with the relativity group.

In order to find the physical content of the general principle of relativity, that is, the covariance properties that are associated in an essential way with the general theory of relativity, Kretschmann follows the same theoretical way: «An attempt will be made to cast it [the general theory of relativity] into the least

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<sup>12</sup> As a consequence of the identification of the notions of 'reference system' and 'coordinate system' one obtains that «between any two reference systems there is a well-defined coordinate transformation, and second, that the set of all coordinate transformations acts as a transformation *group* on the collection of all reference systems», [7], p. 434. Italics in the original.

<sup>13</sup> See [7], p. 441.

covariant form possible without altering its physical content.»<sup>14</sup> Hence one has to fix either all or some of those parts of the determination of the coordinate functions  $g_{\mu}$  which depend solely on the choice of reference system. However, the long and obscure mathematical demonstration does not succeed in finding such a less covariant formulation because of the existence of exceptional cases<sup>15</sup>. Hence, Kretschmann's second attempt is to find it through the analysis of the four-dimensional pictures which represent all the possible motions of light rays and free point masses in a space-time. Then Kretschmann proceeds classifying the geometric pictures into topological equivalence classes [Teilmengen], which constitute the set of all possible solutions for  $g_{\mu}$  in such a way that any two members of the same class are topologically equivalent, and then into subclasses [Untermengen], in such a way that any solution can be obtained from any other of the same subclass by some transformation. In the general case an equivalence class consists of a continuum of subclasses. The idea is to choose a minimal subset of solutions for  $g_{\mu}$  having the same physical content as the entire class of solutions of the covariant equations: select one space-time model from each subclass, for each of which there is a topological coincidence model. Kretschmann's question now, is what is the group which preserves the topological coincidence model: in the general case this is the trivial group of the identity transformations. Hence Kretschmann's conclusion is the statement that the general theory of relativity does not satisfy the principle of relativity: it is a completely absolute theory in regard to its content.

As one can see, what the modern and accepted interpretation retains of the general theory of relativity of Kretschmann's contribution is not much: only the first considerations. But, can we well accept it without further analysis? As Norton<sup>16</sup> pointed out: no. Norton's work is important in reference to the history of the mathematical formulations of the general theory of relativity. He shows that the principle of general covariance was not devoid of physical content in Einstein's formulation: Einstein used number manifolds to represent space-time, introduced much more mathematical structure into the model of the theory than just coordinate systems. Hence, the principle of covariance was necessary to deny physical significance of superfluous structures introduced into space-time. Since only according to the modern formulations is the principle of general covariance devoid of content, one should ask how Kretschmann could reach the same conclusion referring to Einstein's formulation. In the context of Kretschmann's formulation, as well as in that of Einstein, one can say that each equation can be made generally covariant only if one introduces a further contingent physical hypothesis. In Kretschmann's case, as well as in that of Einstein, the physical hypothesis is very meaningful: 'the physical content of a space-time theory is fully exhausted by the catalogue of its space-time coincidences' (point-coincidence argument). Because of the presence of this assumption the principle of covariance has physical significance in the context of Einstein's formulation and, in the final analysis, for the same reason, for Norton, Kretschmann had thus failed to demonstrate the physical vacuity of general covariance.

### **3. V. Fock and the theory of gravitation**

Fock's work was published in the Soviet Union in 1955 and translated into english for the first time in 1961. This work is very important because many

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<sup>14</sup> «...wird man versuchen, sie [die allgemeine relativitätstheorie] ohne Änderung ihres physikalischen Inhaltes auf eine möglichst wenig kovariante Form zu bringen», [5], p. 585.

<sup>15</sup> See [5], 21st and 22nd paragraphs, pp. 601-606.

<sup>16</sup> See [6].

notions, which became part of the subsequent interpretations of the theory of general relativity, are there expressed.

One of the purposes of this work is to demonstrate that there not exists a general principle of relativity and more generally that the theory is not a theory of general relativity but a theory of gravitation.

With reference to Friedman's work and as a general consideration one can say that the possibility of formulating space-time theories in a generally covariant formulation is clearly expressed by Fock without much emphasis. Then, in reference to the principle of equivalence, he introduces what would later become Friedman's purpose to justify a theory of gravitation in the most direct (satisfactory) way. Finally he anticipates some aspects of Norton's analysis of the covariance principle in reference to Kretschmann's objection.

Given the goal of Fock's work, is very important to understand what Fock intends as a principle of relativity: «*a relativity principle is a statement concerning the existence of corresponding processes in a set of reference frames of a certain class*»<sup>17</sup>, that is, of a class of equivalent reference frames. Moreover, two reference frames are called 'physically equivalent' if «*phenomena proceed in the same way in them*», that is, if a possible process is described in the coordinates  $(x)$  by the functions (of state)

$${}_1(x), {}_2(x), \dots, {}_n(x),$$

than there is another possible process which is describable by the same functions (of state)

$${}_1(x), {}_2(x), \dots, {}_n(x)$$

in the coordinates  $(x)$ .

This connections between the principle of relativity and the concept of 'equivalent reference frames' means that we can speak in a sensible way of a 'principle of relativity', if, and only if, we have already clearly specified what kinds of reference frames constitute the class.

Through this definition these principles are clearly thought as principles endowed with physical content.

Since a generally covariant formulation - explicitly necessary if we are in the field of general relativity theory - of the equations describing physical processes needs that the components of the metric tensor  $g_{\mu}$  are included in the functions of state and given the definition of 'equivalent reference frames', then also the components of the metric tensor must have the same mathematical form when we go from one reference frame to another equivalent to the former.

Since the functions of state contain the metric tensor, we have two possibilities: either the metric is fixed (theory of special relativity) or the metric can be affected by the events (theory of general relativity).

In fact, we can consider the equation of wave front propagation, which can be stated in the form:

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = 0$$

which describes a rectilinear propagation of light<sup>18</sup>.

Since light possesses energy and given the law of proportionality of mass and energy, light must possess mass. Moreover, by the law of gravitation, any mass

<sup>17</sup> See [3], p. 179. Italics in original.

<sup>18</sup> One chooses this equation in order to study the properties of space-time because it is one of those of greatest generality and which characterise most directly the properties of space-time.

located in a gravitational field is affected from that field and in general its motion will not be rectilinear, that is, the equation of wave front must have a different form from that given above. But, as we said, this kind of equation expresses the basic properties of space-time. Hence the properties of space-time must be affected from the presence of the gravitational field and its metric cannot be fixed. In conclusion, since the theory of general relativity considers gravitational field, its metric cannot be fixed: events affect the geometrical properties of space-time. But we do not have to forget that this relation between gravitational field and geometrical properties is mutual in Einstein's equations: «On the one hand the deviations of geometrical properties from the Euclidean are determined by the presence of gravitating masses, on the other the motion of masses in the gravitational field are determined by these deviations.»<sup>19</sup>

This connection among metric and gravitation is intuitively justified by the generalisation of Galileo's law that in the absence of resistance all bodies fall equally fast: provided the initial conditions, in a gravitational field all otherwise free bodies move in the same manner. In other words this dynamical generalisation of Galileo's law can be expressed as a statement that the inertial and the gravitational masses of any body are equal.

Fock's treatment of the theory of general relativity - theory of gravitation - is based on the aforementioned dynamical generalisation of Galileo's law, but Einstein's treatment is different: he considered a kinematical consequence of that law. We know now that this kinematical consequence is the principle of equivalence, which, generally expressed, is the statement that in some sense a field of acceleration is equivalent to a gravitational field. In particular, if we introduce a suitable coordinate system, interpreted as an accelerated reference frame, the equations of motion of a mass point in a gravitational field can be transformed into equations of motions of a free point in that new system and, since the values of the gravitational mass and the inertial mass are the same, then this transformation is the same for any value of the mass of the point. The most important difference between the generalisation of Galileo's law and the principle of equivalence concerns their fields of validity: the former is endowed of a generally non-local character, while the latter exists only locally, that is, it only refers to a single point in the space-time. Fock's aim is to demonstrate that the principle of general relativity is not generally valid, because the principle of equivalence does not have a general character. In fact the principle of equivalence would have general validity if the gravitational field is uniform. Since the gravitational field is in general not uniform, the validity of the principle of equivalence can be assumed to hold only in reference to a spatial neighbourhood of the points on a world line, which is of the nature of a time axis. In this sense, the validity of the general principle of relativity is local as well. Moreover, because of the non-uniformity of the gravitational field, not every gravitational fields can be replaced by a field of acceleration.

Furthermore it is still not sure that the principle of general relativity is locally valid. In fact, for Fock, the problem arises because it is not possible to give a clear definition of such accelerated reference systems and, as we have seen above, this is a main point in order to give physical content and validity to a principle of relativity and to understand it. In fact, the famous lift example of Einstein considers a rigid body, which is not acceptable by the theory of general relativity because bodies located in the same field of acceleration experience different deformations from each other. Hence «in Einstein's reasoning the basic concept of a frame of reference in accelerated motion remains undefined.»<sup>20</sup>

With reference to the principle of covariance, Fock means that it is an always satisfiable and self-evident, purely logical requirement. Therefore this

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<sup>19</sup> See [3], p. 189.

<sup>20</sup> See [3], p. 229.



requirement is not necessarily devoid of physical content: a «covariance of coordinate system acquires a definite physical meaning if, and only if, a principle of relativity exists for the class of reference frames used.»<sup>21</sup> In fact, although the principle of general relativity implies a covariance of the equations, thereby: covariance of differential equations is possible also when no principle of relativity is satisfied and, moreover, there exist some laws which cannot be expressed in a differential form.

Furthermore, as we have seen in Norton's analysis, the requirement of covariance can be satisfied only if further hypothesis are made. In the case of Fock is not precisely a matter of hypothesis but of initial and boundary conditions: each field described using differential equations, requires for its definition also all kind of other conditions, which are not covariant. Hence it can happen that the preservation of a physical content requires a change of its mathematical form and *vice versa*. In ultimate analysis the realizability of a process with a given physical content in different coordinate systems is a question which cannot be solved *a priori*. Moreover, if such a process is possible in different reference frames, then there exists a principle of relativity and the principle of covariance acquires physical content.

#### **4. H. Bondi and the non-uniformity of the gravitational field**

Fock's issue in reference to the non-uniformity of the gravitational field is resumed by H. Bondi. The central point of Bondi's argumentation is represented by the falsification of the principle of equivalence in reference to wide spatial region (however small). We consider as Bondi's starting point the definition of 'observable in a gravitational field': that is, «the relative acceleration of neighbouring particles»<sup>22</sup>, or, in the terms in use in the theory of general relativity, the 'geodesic deviation'.

Hence a field, which is defined by accelerated particles contained in that field, is a non-uniform field by four reasons:

- 1) because the acceleration can be characterised by different directions;
- 2) since the relative acceleration decreases as soon as the particles approach one another, then the non-uniformity is also due to the gap between the particles in a directly proportional way;
- 3) because the value of relative acceleration can vary in consequence of the velocity: in the special theory of relativity the acceleration depends necessarily on the velocity because, otherwise, it would be logically possible a system of particles, in which one or some of these moves with velocity higher than  $c$ ;
- 4) because the value of relative acceleration can vary with time and this because the natural gravitational field is not eliminable. In fact, unlike any other property of a body, the gravitational mass is given in nature as an not-abolishable property, hence the gravitational field is not eliminable. Moreover, by the law of conservation of momentum, masses (sources, bodies) must have a motion. Since the motion of bodies involves a shift of frequency of the radiated energy, and since time, like any other physical quantity, is defined by the way in which it is measured, that is, the frequency of light (in reference to the best measuring instruments), then the time slows down proportionally in consequence of decreasing of distance between the observer and the gravitational source.

In conclusion, the principle of equivalence can not be extended to finite regions.

#### **Conclusion**

One of the now accepted interpretations of the theory of general relativity (Friedman's interpretation) has a long history, which begins simultaneously with

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<sup>21</sup> See [3], p. 182.

<sup>22</sup> See [1], p. 115.

the issue of Einstein's theory. The first step is represented on the reflection about the principle of general covariance and, in particular, on the statement that this principle is devoid of physical content, although the first attempt in order to demonstrate it (Kretschmann's attempt) failed. The second step, moreover, regards the principle of equivalence: it has neither general nor local validity but only "infinitesimal", and, since this principle represents the physical idea that should hold the general principle of relativity, one can conclude that the latter is not valid and that the theory is not a very theory of relativity.

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